Mini-Course Description for MTNS 2014: **From Sampled-data Control to Signal Processing**

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Overview and Background

There has been remarkable progress in sampled-data control theory in the past two decades. The main achievement here is that there exists a digital (discrete-time) control law that takes the intersample behavior into account and makes the overall analog (continuous-time) performance optimal, in the sense of H^∞-norm. This naturally suggests its application to digital signal processing where the same hybrid nature of analog and digital is always prevalent. A crucial observation here is that the perfect band-limiting hypothesis, widely accepted in signal processing, is often inadequate for many practical situations. In practice, the original analog signals (sounds, images, etc.) are neither fully band-limited nor even close to be band-limited in the current processing standards. Sampled-data control theory provides an ideal platform for dealing with these problems, and the new design method based on sampled-data control has become highly successful in commercial applications of sound processing chips (45 million chips produced to date) and AAC/MP3 sound processing audio players applications (iPhone/iPod App).

This mini-course intends to provide the following:

1. Introductory overview of modern sampled-data control theory; in particular, we carefully review
    - the difficulties in dealing with sampled-data systems,
    - how lifting reduces a sampled-data control system into a linear, time-invariant, discrete-time system,
    - how this reduction recovers such classical notions of transfer operators, steady-state response, and frequency response, etc., and

- how an H^∞ sampled-data control problem can be reduced to a discrete-time H^∞ problem.

2. After covering such fundamental issues, we review a basic problem of signal reconstruction in digital signal processing. It is noted that the current paradigm due to Shannon yields various difficulties in practice, and we give an overview of such problems. We then turn our attention to the application of sampled-data control theory to signal processing.

3. We will review how the above signal reconstruction problem is reducible to a sampled-data H^∞ control problem, and how it can be solved. We will give an overview of the varied advantages of this present method over the conventional methods using several sound and image processing examples. In the final section, we will give some comparisons with current approaches on non-ideal sampling theory, and show the advantages of the present method based on sampled-data control theory.

Course Outline and Time Schedule

We will give this course in two parts:

Part A (2 slots, 50 min) Basics in Sampled-data Control Theory

Part B (3 slots, 75 min) Application to Signal/Image/Video Processing

The following are the detailed descriptions of the parts.

Part A: Basics in Sampled-data Control Theory

This part consists of two 2 25-min slots, and covers some basics in modern sampled-data control theory. The main difficulty here is that sampled-data systems are not time-invariant in an ordinary sense, due the mixture of two different time-sets. This yields various practical difficulties. The lack of steady-state response, frequency response, or even transfer functions is a prime evidence. As is now well recognized, the lifting technique has played a key role in resolving this fundamental difficulty.

The following topics will be covered:

1st 25-min slot:

- Fundamental difficulty in sampled-data control—problems arising from the lack of time-invariance.
- Introduction of lifting and recovery of time-invariance.
- Closed-loop equations and transfer operators.
- Steady-state response and frequency response for sampled-data systems.

2nd 25-min slot:

- H^∞ (and H^2) control problems for sampled-data systems.
- Their reduction to finite-dimensional problems.
- Design examples.

Part B: Application to Signal/Image/Video Processing

Digital signal processing shares the same difficulty of the mixture of continuous and discrete data. In dealing with the problem of signal recovery from sampled-data, i.e., signal reconstruction problem, Shannon introduced the well-known Shannon paradigm based on the sampling theorem. That is, one assumes that the original analog signal must be completely band-limited in the frequency domain below the so-called Nyquist frequency.

But the current digital signal processing applications do not allow this basic assumption to hold even approximately, and various difficulties and distortions arise from this. We will present how this problem can be resolved using sampled-data H^∞ control theory, and also show that this new theory possess various advantages over the classical Shannon paradigm. We will also give some comparisons with the current approaches on non-ideal sampling theory, and show that the present method has some advantages not shared by these approaches. The new method has proven to be successful in that it is now implemented in sound-processing chips by Sanyo semiconductor, and the total production has reached 45 million chips. There is also an application to iPhone/iPod music players.

The following topics will be covered in this second part:

3rd 25-min slot:

- Overview of various difficulties in the band-limiting signal processing.
- Formulation of the signal reconstruction problem as a sampled-data control problem.
- Solution to the reduced sampled-data problem.

4th and 5th 25-min slots:

- Comparison with non-ideal sampling theories by Unser, Eldar and others.
- Examples and sound/image/video demonstrations:
  - Audio signal processing
  - Image/video super-resolution
  - **[a brief break]**
  - Voice restoration in mobile phones
  - Image denoising with nonlinear processing
  - Real-time video denoising
  - Fractional delay filters and sound synthesis

Background for the Course

No background for sampled-data theory or signal processing is required, although some familiarity with robust and H^∞ control theory is expected.